Spearman's Correlation

Spearman's ρ (rho) is a non-parametric measure of correlation between two ranked variables. It is similar to Pearson's Product Moment Correlation Coefficient, or Pearson's r. It indicates magnitude and direction of the association between two variables that are on interval or ratio scale. Like Pearson's r a negative correlation indicates that when X is increasing then Y is decreasing. With Spearman's ρ when there are no ties in the ranking then the correlation is defined in Equation 1.

$$\rho = 1 - \frac{6 \cdot \sum d^2}{n \cdot (n^2 - 1)} \tag{1}$$

Where:

n = number of rank pairs

d = difference between paired ranks

Table 1: Spearman's Rho Rank Correlation Without Ties

Rank 1	Rank 2	Rank1-Rank2	$(Rank1 - Rank2)^2$
1	1	0	0
2	3	-1	1
3	6	-3	9
4	2	2	4
5	7	-2	4
6	4	2	4
7	5	2	4
			26

$$\rho = 1 - \frac{6 \cdot \sum d^2}{n \cdot (n^2 - 1)} = 1 - \frac{6 \cdot 26}{7 \cdot (49 - 1)} = 1 - .46429 = .53571 \tag{2}$$

There are a couple of different options when there are ties in the ranks. One common approach is to use the average of the tied ranks. For example if the tied ranks correspond to 5 and 6 then the average rank becomes 5.5. Table 2 shows the same data from Table 1 but it has tied ranks for Rank 2. Equation 3 shows the correlation for the data using tied ranks.

$$\rho = 1 - \frac{6 \cdot \sum d^2}{n \cdot (n^2 - 1)} = 1 - \frac{6 \cdot 21.5}{7 \cdot (49 - 1)} = 1 - .384 = .616 \tag{3}$$

Table 2: Spearman's Rho Rank Correlation With Ties

Rank 1	Rank 2	Adjusted Rank 2	Rank1-Rank2	$(Rank1 - Rank2)^2$
1	1	1	0	0
2	3	3	-1	1
3	6	5.5	-2.5	6.25
4	2	2	2	4
5	7	7	-2	4
6	4	4	2	4
7	6	5.5	1.5	2.25
				21.5

The process to calculate Spearman's ρ is quite simple using the R Environment.

```
library(Hmisc);
##No Ties
x <- c(1,2,3,4,5,6,7);
y <- c(1,3,6,2,7,4,5);
rcorr(x,y,type="spearman");
##Ties
x2 <- c(1,2,3,4,5,6,7);
y2 <- c(1,3,6,2,7,4,6);
rcorr(x2,y2,type="spearman");
```

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